

# Intermediate mass black holes in AGN disks I. Production & Growth

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## ABSTRACT

Here we propose a mechanism for efficiently growing intermediate mass black holes (IMBH) in disks around supermassive black holes. Stellar mass objects can efficiently agglomerate when facilitated by the gas disk. Stars, compact objects and binaries can migrate, accrete and merge within disks around supermassive black holes. While dynamical heating by cusp stars excites the velocity dispersion of nuclear cluster objects (NCOs) in the disk, gas in the disk damps NCO orbits. If gas damping dominates, NCOs remain in the disk with circularized orbits and large collision cross-sections. IMBH seeds can grow extremely rapidly by collisions with disk NCOs at low relative velocities, allowing for super-Eddington growth rates. Once an IMBH seed has cleared out its feeding zone of disk NCOs, growth of IMBH seeds can become dominated by gas accretion from the AGN disk. However, the IMBH can migrate in the disk and expand its feeding zone, permitting a super-Eddington accretion rate to continue. Growth of IMBH seeds via NCO collisions is enhanced by a pile-up of migrators.

We highlight the remarkable parallel between the growth of IMBH in AGN disks with models of giant planet growth in protoplanetary disks. If an IMBH becomes massive enough it can open a gap in the AGN disk. IMBH migration in AGN disks may stall, allowing them to survive the end of the AGN phase and remain in galactic nuclei. Our proposed mechanisms should be more efficient at growing IMBH in AGN disks than the standard model of IMBH growth in stellar clusters. Dynamical heating of disk NCOs by cusp stars is transferred to the gas in a AGN disk helping to maintain the outer disk against gravitational instability. Model predictions, observational constraints and implications are discussed in a companion paper (Paper II).

**Key words:** galaxies: active – (stars:) binaries:close – planets-disc interactions – protoplanetary discs – emission: accretion

## 1 INTRODUCTION

Extensive evidence exists that supermassive black holes ( $> 10^6 M_\odot$ ) are found in the centers of most galaxies (e.g. Kormendy & Richstone 1995). Extensive evidence also exists for stellar mass black holes in our own Galaxy (Remillard & McClintock 2006). Stellar mass black holes are expected to form as the end product of high-mass stars. Supermassive black holes, by contrast, have grown

to their current size over cosmic time, from much smaller seeds (e.g. Begelman & Rees 1978; Islam, Taylor & Silk. 2004; Portegies-Zwart et al. 2004; Micic et al. 2011, & references therein). Intermediate mass black holes (IMBH;  $\sim 10^2 - 10^4 M_\odot$ ) may have been the original seeds for supermassive black holes or, they may have contributed to fast early growth of such seeds via mergers (e.g. Madau & Rees 2001; Miller & Colbert 2004). Though we expect IMBH should exist, at least as an intermediate stage on the way to a supermassive black hole, observationally the evidence for their existence is scant and ambiguous, especially compared with

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evidence for supermassive and stellar mass black holes. The low mass end of the supermassive black hole distribution in galactic nuclei may extend down to  $\sim 10^5 M_\odot$  (Jiang et al. 2011), but below this mass the evidence becomes ambiguous. The ultra-luminous X-ray sources (ULXs) observed outside galactic nuclei (e.g. Winter et al. 2009) may be powered by accretion onto IMBH (Miller & Colbert 2004). However ULXs could also be explained by beamed radiation from accreting stellar-mass black holes (King 2009) and power-law dominated ULXs might be due to background AGN. IMBH have so far been hard to find and constrain in the local Universe, either in our own Galaxy or at low  $z$ .

Active galactic nuclei (AGN) are believed to be powered by accretion onto a supermassive black hole. The accretion disk should contain a population of stars and compact objects (collectively nuclear cluster objects, NCOs) that can migrate within and accrete from the disk (e.g. Ostriker 1983; Syer, Clarke & Rees 1991; Artymowicz et al. 1993a; Goodman & Tan 2004; Levin 2007; Nayakshin & Sunyaev 2007; McKernan et al. 2011a,b). In McKernan et al. (2011a) we speculated that IMBH seeds may form efficiently in AGN disks due to NCO collisions and mergers, which is quite different from the standard model of stellar mass black holes merging in stellar clusters (e.g. Miller & Hamilton 2002; Miller & Colbert 2004). Here we argue that IMBH production is in fact far more likely and more efficient in AGN disks, with implications for AGN observations, duty cycle and supermassive black hole accretion rates.

In this paper (and its companion, Paper II, McKernan et al. 2012) we discuss semi-analytically the production of intermediate mass black holes in the environment of AGN disks. Discussion of observational predictions of this model of IMBH growth as well as consequences for AGN disks, duty cycles and the demographics of activity in galactic nuclei at low and high redshift will be left to Paper II. In section §2, we discuss why we think IMBH can be built in AGN disks. In section §3 we explore mechanisms that will be important in actually growing IMBH in AGN disks, including the competing forces of eccentricity damping and excitation in the disk. The importance of IMBH migration is outlined in section §4. Section §5 outlines a simple model of IMBH growth in AGN disks and we highlight the remarkable parallel between the growth of IMBH in AGN disks and the growth of giant planets in protoplanetary disks. Finally in section §6, we outline our conclusions and future work.

## 2 WHY IMBH CAN BE BUILT IN AGN DISKS

The largest, supermassive, black holes in the Universe ( $M_{\text{BH}} \sim 10^6 - 10^9 M_\odot$ ) live in galactic centers (e.g. Kormendy & Richstone 1995). We expect a dense nuclear cluster of objects to surround the supermassive black hole as a result of stellar evolution, dynamical friction, secular evolution and minor mergers (e.g. Morris 1993; Miralda-Escudé & Gould 2000; Merritt 2010). In our own Galaxy, the distributed mass within  $\sim 1$  pc of Sgr A\* is  $\sim 10 - 30\%$  of the mass of the supermassive black hole (Schödel et al. 2009). If a large quantity of gas somehow arrives in the innermost pc of a galactic nucleus (e.g. Krolik 1999; Kawawatu et al. 2003; Vittorini et al. 2005; Hopkins & Hernquist 2006; McKernan et al. 2010b), it will

likely lose angular momentum and accrete onto the central supermassive black hole. But in doing so, gas must also interact with the NCO population. Depending on the aspect ratio of the disk that forms, a few percent of NCO orbits are likely to coincide with the accretion flow. A small percentage of NCO orbits coincident with the geometric cross-section of a thin disk would lead to an initial population of  $\sim 10^3 M_\odot$  of NCOs in a pc-scale accretion flow around a SgrA\* sized black hole.

NCOs can exchange angular momentum with gas in the disk, and each other, so they can scatter each other and migrate within the disk (see McKernan et al. 2011a). The processes involved are analogous to protoplanetary disk theory (e.g. Pollack et al. 1996; Armitage 2010). Indeed, physical conditions in the *outskirts* of AGN disks are relatively close to those in protoplanetary disks (McKernan et al. 2011a). The migration of NCOs in the disk will enhance the probability of collisions, mergers and ejections. Under these conditions, IMBH seeds can grow. IMBH seeds will be objects  $\geq 10 M_\odot$  that will not lose very much mass (e.g. stellar mass black holes, hard massive binaries). IMBH 'seedlings' we define as objects  $\geq 10 M_\odot$  that have grown via mergers (e.g. the merged end-product of a hard binary). IMBH seeds and seedlings located at semi-major axis  $a$  in an AGN disk will maintain a 'feeding zone' within which they may collide with nearly co-orbital disk NCOs. By analogy with proto-planet growth we define the feeding zone to be  $a \pm 4R_H$  where  $a$  is the IMBH semi-major axis and  $R_H = a(q/3)^{1/3}$  is the IMBH Hill radius, with  $q$  the mass ratio of IMBH:supermassive black hole. Once the object gets to  $\geq 100 M_\odot$  we will call the result an IMBH.

The disk NCO population is subjected to dynamical heating from cusp stars and dynamical cooling from gas damping. If gas damping dominates, IMBH seeds and disk NCOs will have their orbits rapidly damped. As a result their collision cross-sections will rapidly increase (since the relative velocity of encounters will be small, particularly in the outer disk). IMBH seedlings will initially accrete disk NCOs within their feeding zone in a 'core accretion' mode of growth. Once nearby NCOs have been scattered or accreted, gas accretion dominates IMBH growth. However, the migration of IMBH seedlings within the disk allows growth to continue via collisions as well as via gas accretion. Thus, we expect IMBH to grow within AGN disks, analogous to the growth of giant planets within protoplanetary disks and we expect the IMBH growth rate will be much larger than in stellar clusters.

Here we concentrate on growing IMBH within the AGN disk itself. Of course it is possible that IMBH already exist in the galactic nucleus when the AGN disk first forms. A top-heavy initial mass function of cusp stars can lead to IMBH seedling formation before low angular momentum gas arrives in the nucleus. IMBH can also arrive from outside the galactic nucleus to interact with the AGN disk since mass segregation and dynamical friction can deliver IMBH to the central parsec of galactic nuclei in a few Gyrs from nearby clusters (e.g. McKernan et al. 2011b, see also Paper II).

The physics involved in IMBH formation in AGN disks spans multiple regimes and physical processes and would usefully benefit from detailed numerical simulations. Such simulations require realistic treatments of (amongst others): N-body collisions, mergers, accretion,

tidal forces, gravitational radiation, Special & General Relativity, radiative transfer, the magneto-rotational instability and the gravitational instability. However at present there are no simulations that can adequately address the relevant physics in a self-consistent manner. We take a semi-analytic approach following (e.g. Miller & Hamilton 2002) on the build-up of IMBH in star clusters, (e.g. Alexander, Begelman & Armitage 2007) on stellar dynamical heating and cooling, as well as formalism on planet growth from protoplanetary theory (e.g. Pollack et al. 1996; Armitage 2010, & references therein).

### 3 HOW TO BUILD IMBH IN AGN DISKS

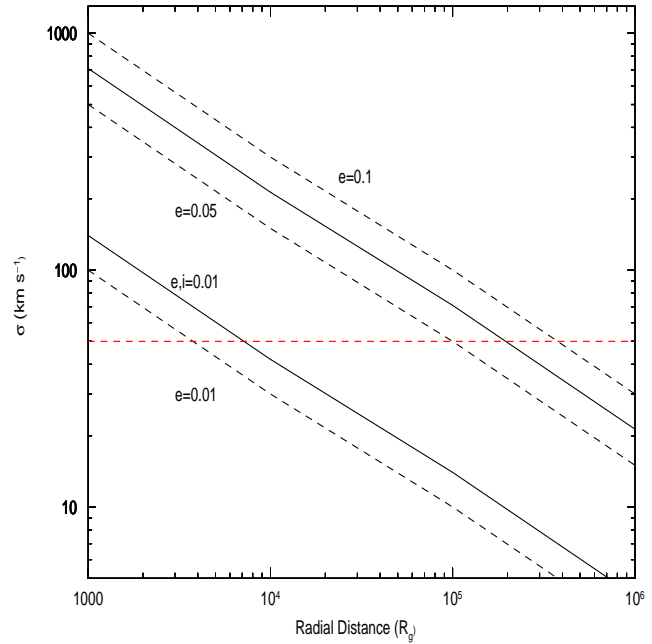
In this section, we shall outline the key phenomena involved in growing IMBH seeds in AGN disks. In order to grow into IMBH, seeds must collide with and accrete mass, either NCOs or gas. In section §3.1 we discuss NCO collision cross sections in AGN disks and the importance of eccentricity damping and excitation. In section §3.2 we outline a model of dynamical heating and cooling of NCO orbits in AGN disks and we discuss the implications for IMBH seed growth. Section §3.3 briefly outlines issues involved in merging binaries, a potentially important channel for producing IMBH seedlings.

#### 3.1 Collision cross-sections in the disk

In the absence of disk gas, NCOs change their orbits only due to weak gravitational interactions, occurring on the (long) relaxation timescale (see below). The interaction with a gaseous disk gives rise to new effects, namely torques. NCOs embedded in the disk, or crossing it, will have their eccentricities and inclinations damped relative to the disk. Such processes should enhance the stellar density in the disk region and lower the velocity dispersion of NCOs embedded in the disk, in the absence of other important effects. This, in turn, gives rise to a higher rate of encounter and collisions between NCOs in the disk. The collisional cross-section ( $\sigma_{\text{coll}}$ ) of compact NCOs of mass  $M$  depends on the relative velocity at infinity ( $v_\infty$ ) as

$$\sigma_{\text{coll}} \approx \pi r_p (2GM/v_\infty^2) \quad (1)$$

in the gravitational focussing regime, where  $r_p$  is the separation at periastron. In AGN disks the *relative* velocities involved in close encounters can be very small compared to the velocity dispersion in star clusters (typically  $\sim 50\text{km/s}$ ). For example, NCOs on circularized orbits separated in the disk by  $\Delta R \sim 0.01R$  at  $R = 10^5 r_g$  have relative velocities due to Keplerian shear at periastron of only  $\sim 5\text{kms}^{-1}$ , where  $r_g = GM_{\text{SMBH}}/c^2$  is the gravitational radius of the supermassive black hole of mass  $M_{\text{SMBH}}$ . The disk NCO velocity dispersion varies with radius as  $\sigma \approx \sqrt{\bar{e}^2 + \bar{i}^2} v_k$ , where  $\bar{e}, \bar{i}, v_k$  are the mean NCO orbital eccentricity, mean NCO orbital inclination and Keplerian velocity respectively. Fig. 1 shows the NCO velocity dispersion ( $\sigma$ ) as a function of radius in a Keplerian AGN disk for a range of mean eccentricities and inclinations. Also shown in Fig. 1 is the typical velocity dispersion in star clusters ( $\sim 50\text{km s}^{-1}$ , red horizontal dashed line). So  $v_\infty$  for a typical interaction in



**Figure 1.** The velocity dispersion ( $\sigma \approx \sqrt{e^2 + i^2} V_{\text{kep}}$ ) of NCOs in a Keplerian AGN disk. Shown are  $\sigma$  as a function of disk radius, for eccentricity and inclination values of  $(e,i) = 0.01, 0.05$  (solid lines) and  $e = 0.01, 0.05, 0.1$ , with  $i = 0$  (dashed lines). Also shown (red dashed horizontal line) is the typical velocity dispersion in star clusters ( $\sim 50\text{ km s}^{-1}$ ). Note that most disk NCOs should live in the outer disk ( $> 10^4 r_g$ ) for an NCO population that grows as  $r^2$ .

a stellar cluster is  $\sim 50\text{km s}^{-1}$ . Fig. 1 shows that the velocity dispersion of NCOs at large disk radii is less than in star clusters for small to moderate NCO orbital eccentricities and inclinations ( $e, i \sim 0.01 - 0.05$ ). Since the numbers of NCOs should increase with radius, most NCOs should live in the outer disk, where the NCO velocity dispersion should be smallest.

The collisional cross-section of a seed IMBH (mass  $M$ ) with compact objects (mass  $m$ ) such as neutron stars, white dwarfs and stellar mass black holes depends on relative velocity as (Quinlan & Shapiro 1989)

$$\sigma_{\text{coll}} = 2\pi \left( \frac{85\pi}{6\sqrt{2}} \right)^{2/7} \frac{G^2 m^{2/7} M^{12/7}}{c^{10/7} v_\infty^{18/7}}. \quad (2)$$

Numerically, this can be written as  $\sigma_{\text{coll}} \approx 2 \times 10^{26} m_{10}^{2/7} M_{50}^{12/7} v_6^{-18/7} \text{cm}^2$  where  $v_\infty = 10^6 v_6 \text{cm s}^{-1}$  and  $M_{50}, m_{10}$  are in units of  $50M_\odot, 10M_\odot$  respectively (Miller & Hamilton 2002). For  $M_{50}, m_{10} = 1$ , located  $\sim 10^5 r_g$  from a supermassive black hole, small eccentricities  $e \sim 0.01$  in Fig. 1 lead to collision cross-sections up to an order of magnitude larger than in clusters (for  $M_{50}, m_{10} = 1; v_6 = 5$  above). However, for large NCO orbital eccentricities ( $e \geq 0.1$ ), IMBH collisions in AGN disks will have smaller cross-sections than in star clusters, over most of the disk. Therefore if the initial mean eccentricity ( $\bar{e}$ ) of the disk NCO distribution is large, mechanisms for damping orbital eccentricities will be very important in determining whether IMBH growth via collisions is efficient in AGN disks.

### 3.2 NCO orbital damping & excitation

We begin with a fully analytic approach, demonstrating the relative importance of competing terms and effects. The velocity dispersion ( $\sigma$ ) of NCOs in a disk is excited by dynamical heating and is damped by dynamical cooling. Thus

$$\frac{d\sigma}{dt} = \Delta Q_+ - \Delta Q_- \quad (3)$$

where  $\Delta Q_+$  is the dynamical heating term and  $\Delta Q_-$  is the dynamical cooling term. Dynamical heating comes from two sources: the relaxation of disk NCOs through mutual interactions and the dynamical excitation of disk NCOs by cusp NCOs, so

$$\Delta Q_+ = \delta Q_{\text{relax}} + \delta Q_{\text{excite}}. \quad (4)$$

Considering first the relaxation term, we assume that there are  $N_1$  stars of mass  $M_1$  and velocity dispersion  $\sigma_1$  in an annulus of width  $\Delta R$  centered on  $R$ . The relaxation timescale is given by (Alexander, Begelman & Armitage 2007)

$$t_{\text{relax}} = \frac{2\pi C_1 R \Delta R \sigma_1^4}{G^2 N m^2 \ln \Lambda \Omega} \quad (5)$$

so

$$\delta Q_{\text{relax}} = \frac{\sigma}{t_{\text{relax}}} = \frac{D_1}{C_1} \frac{1}{\sigma_1^3} \quad (6)$$

where

$$D_1 = \frac{G^2 N_1 M_1^2 \ln \Lambda_1}{R \Delta R t_{\text{orb}}} \quad (7)$$

where  $\Omega$  is the Keplerian frequency,  $\ln \Lambda_1$  ( $\sim 9$ ) is the Coulomb logarithm and  $C_1 \sim 2.2$ . The solid curve in Fig. 2 shows the evolution of  $\langle e^2 \rangle^{1/2}$  due to relaxation alone for a population of  $N_1 = 10^3$  stars of mass  $M_1 = 0.6M_\odot$  in an annulus of width  $\Delta R = 0.1\text{pc}$  centered on  $R = 0.1\text{pc}$  (equivalently  $1 - 3 \times 10^4 r_g$  of an AGN disk around a  $10^8 M_\odot$  supermassive black hole). For these stars,  $v_k = 2100\text{km/s}$  and so  $t_{\text{orb}} = 9 \times 10^9\text{s}$ . Since, for moderate eccentricities,

$$\sigma = \frac{\langle e^2 \rangle^{1/2} v_k}{\sqrt{2}} \quad (8)$$

the solid curve in Fig. 2 follows a  $t^{1/4}$  form and rises (limited by the increase in  $\sigma$  to approximately the Keplerian velocity). This solid curve applies to an isolated annulus of stars in the absence of competing effects.

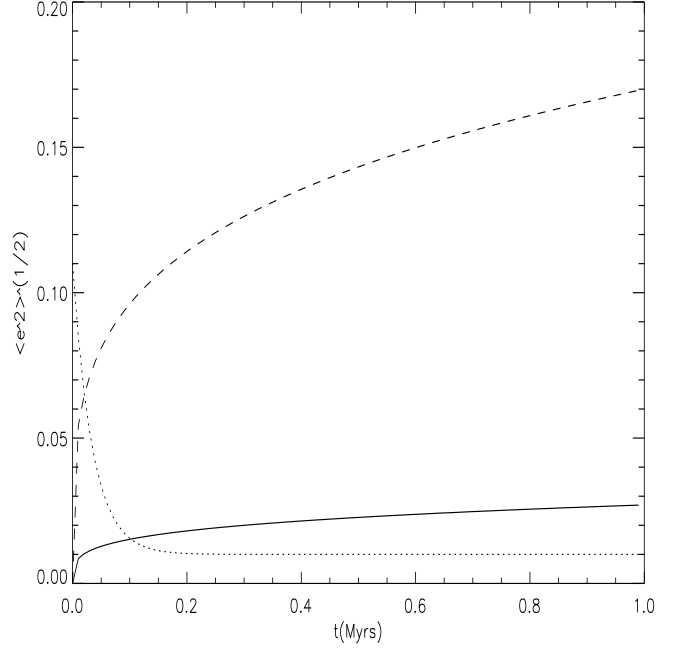
Additional heating is supplied by the cusp population. The cusp stars will transfer kinetic energy to the 'colder' disk population and excite the  $\sigma_1$  distribution of the disk NCOs (see Perets (2008) and Perets et al., 2012, in prep.). Following Alexander, Begelman & Armitage (2007)  $\delta Q_{\text{excite}}$  has the form

$$\delta Q_{\text{excite}} = \frac{\sigma}{t_{\text{excite}}} = \frac{D_2}{C_2} \frac{\sigma_1}{\bar{\sigma}_{1,i}^4} \left(1 - \frac{E_1}{E_i}\right) \quad (9)$$

where the cusp population  $N_i \gg N_1$  has a similar mass function  $M_i = M_1$  to the NCOs in the disk and

$$D_2 = \frac{G^2 N_i M_i M_1 \ln \Lambda_{1i}}{R_i \Delta R t_{\text{orb}}} \quad (10)$$

with  $\bar{\sigma}_{1,i} = (\sigma_1 + \sigma_i)/2$  and  $E_{1,i} = 3M_{1,i}\sigma_{1,i}^2$  is the kinetic energy. Note that  $t_{\text{excite}}$  is analogous to the  $t_{\text{relax}}$  term in equation 5 except  $N$  is now the cusp population ( $N_i$ ). So



**Figure 2.** The rms eccentricity as a function of time for an annulus of  $10^3$  stars of identical mass  $0.6M_\odot$  located in an AGN disk between  $0.05\text{--}0.15\text{pc}$  (or  $1\text{--}3 \times 10^4 r_g$ ) around a  $10^8 M_\odot$  supermassive black hole.  $\langle e^2 \rangle^{1/2}$  scales as  $N^{1/4}, M^{1/2}$ . The solid curve shows the relaxation of the isolated distribution of stars assuming initial circularized orbits. The dashed curve shows relaxation *plus* dynamical heating from a population of  $N_i = 100N_1$  cusp stars with  $M_i = M_1$  and  $\bar{\sigma}_i \sim 0.5v_k$ . The dotted line shows the net domination of exponential damping by disk gas over orbital excitation by cusp stars.

$t_{\text{excite}} \sim (N_1/N_i)t_{\text{relax}} \ll t_{\text{relax}}$ . We assume that on average  $\bar{\sigma}_i \sim \sqrt{e^2 + i^2} v_k \sim 0.5v_k$  so  $E_i > E_1$  (i.e. the cusp stars have greater kinetic energy than the disk stars). The dashed curve in Fig. 2 shows the addition of this excitation term to the evolution of  $\langle e^2 \rangle^{1/2}$ , assuming  $N_i = 10^2 N_1$  and  $C_1/C_2 = 3.5$  (Alexander, Begelman & Armitage 2007), with  $\ln \Lambda_{1i} \sim \ln \Lambda_1$ . Clearly, the curve retains a  $t^{1/4}$  dependence, but at larger values of  $\langle e^2 \rangle^{1/2}$ . Thus, dynamical heating by stars in the cusp dominates relaxation by the stars in the disk (see also Perets 2008).

The competing dynamical cooling term  $\Delta Q_-$  is dominated by gas damping of the disk NCO orbits. Gas drag in AGN disks will tend to reduce small NCO orbital eccentricities and inclinations to much smaller values. Gas at co-orbital Lindblad resonances will damp (e,i) for NCOs with  $q \leq 10^{-3}$  (e.g. Artymowicz et al. 1993b; Ward & Hahn 1994; Cresswell et al. 2007; Bitsch & Kley 2010). Since this mechanism depends on the co-rotating gas mass, both stellar and compact NCOs and IMBH seeds will have their orbits damped, particularly in the outer disk where most of the disk mass is located. For small eccentricities ( $e < 2(H/r)$ ), orbital eccentricity decays exponentially over time  $\tau_e \approx (H/r)^2 \tau_{\text{mig}}$ , where  $h = H/r$  is the disk aspect ratio and  $\tau_{\text{mig}}$  is the migration timescale (Ward & Hahn 1994; Bitsch & Kley 2010). Thus

$$\frac{de}{dt} = -\kappa e \quad (11)$$

which will give us a term linear in  $\sigma$  as the damping term in  $\Delta Q_-$ . By analogy with the relaxation term ( $\sigma/t_{\text{relax}}$ ) above, we choose  $\kappa = 1/t_{\text{damp}}$ . The damping timescale is given by (e.g. Horn et al. 2012)

$$t_{\text{damp}} = \frac{M_{\text{BH}}^2 h_{\text{gas}}^4}{m \Sigma a^2 \Omega} \quad (12)$$

where  $h_{\text{gas}} = H/R = c_s/v_k$  is the *gas* disk aspect ratio (the disk of stellar NCOs has an aspect ratio  $h_{\text{stars}} = \sigma/v_k$  but the disk NCOs are not the source of damping). For larger eccentricities ( $e > 2(H/r)$ ) the eccentricity damping goes as (Bitsch & Kley 2010)

$$\frac{de}{dt} = -\frac{\kappa}{e^2} \quad (13)$$

where we choose the same normalization  $\kappa = 1/t_{\text{damp}}$  as above. So,

$$\Delta Q_- = -\kappa \left[ \beta' \sigma + \frac{\beta''}{\sigma^2} \right] \quad (14)$$

where  $\beta' = 1$  if  $e < 0.1$ , zero otherwise and  $\beta'' = 1$  if  $e > 0.1$ , zero otherwise. Thus, our expression for the combined relaxation, excitation and gas damping of the velocity dispersion for an annulus of NCOs is given by

$$\frac{d\sigma}{dt} = \frac{D_1}{C_1} \frac{1}{\sigma_1^3} + \frac{D_2}{C_2} \frac{\sigma_1}{\sigma_1^4} \left( 1 - \frac{E_1}{E_i} \right) - \kappa \left[ \beta' \sigma + \frac{\beta''}{\sigma^2} \right]. \quad (15)$$

For  $< e^2 >^{1/2} < 2h$  (Bitsch & Kley 2010), eqn. 15 has the general form

$$\frac{d\sigma}{dt} = \left[ \frac{\sigma}{t_{\text{relax}}} + \frac{\sigma}{t_{\text{excite}}} \right] - \frac{\sigma}{t_{\text{damp}}} = \frac{A}{\sigma^3} - \kappa \sigma. \quad (16)$$

where we have combined the dynamical heating terms into a single general  $A/\sigma^3$  term. Since  $t_{\text{excite}} \ll t_{\text{relax}}$ , at steady state,  $d\sigma/dt = 0$ , and so  $t_{\text{damp}} \sim t_{\text{excite}}$  and  $e$  takes the general form

$$e^4 \sim \frac{4G^2 N_c m M_{\text{BH}}^2 h_{\text{gas}}^4 \ln \Lambda}{2\pi C_2 \Sigma a^2 R \Delta R v_k^4} \quad (17)$$

where  $N_c$  is the number of stars in the cusp. So, for a population of  $10^3 \times 0.6 M_\odot$  stars located at  $1 - 3 \times 10^4 r_g$  in an AGN disk, with  $h_{\text{gas}} \sim 10^{-2}$  and a cusp population of  $N_c = 10^5$  stars, equilibrium eccentricity is  $e \sim 0.01$ . In Fig. 2 we plot (dotted line) the evolution of  $e$  over time assuming  $i \sim 0$ . From this we see that disk NCOs should rapidly settle down to near circular orbits ( $< e^2 >^{1/2} \sim 0.01$ ) within  $\sim 0.1 \text{ Myr}$ . Therefore collision cross-sections ( $\sigma_{\text{coll}}$ ) of IMBH seeds in AGN disks should rapidly become much larger than typical collision cross sections in star clusters and it is gas damping that makes the difference.

We expect gas damping to become even more dominant if the NCO disk population declines ( $\dot{N}_-$ ) due to mergers, accretion and scatterings. From equipartition of energy, we expect  $\dot{N}_-$  will be dominated by low mass stars. At moderate inclinations, these NCOs can be captured fairly quickly by the disk again (such that  $\dot{N}_+$  increases), if the gas damps the orbital inclination efficiently (Artymowicz et al. 1993a). As  $\dot{N}_+$  increases, the system is driven towards a dynamical equilibrium when  $\dot{N}_- \approx \dot{N}_+$ .

One important point to note from the above discussion is that the dynamical heating of the NCOs by cusp stars

( $\Delta Q_+$ ) gets transferred to the AGN disk *gas*. The stability of the outskirts of the AGN disk is a well-known and unsolved problem (e.g. Sirko & Goodman 2003); dynamical heating of disk NCOs by cusp stars is a new, additional source of disk heating which will contribute to maintaining the outer disk against gravitational instability. A self-consistent calculation of the disk heating requires a disk model (e.g. Sirko & Goodman 2003) and is beyond the scope of this paper, but see McKernan et al. 2012 (in prep.). Nevertheless, we can see that a large density of NCOs in a galactic nucleus will strongly excite the orbits of disk NCOs ( $\delta Q_{\text{excite}}$  is large). Gas damping ( $\Delta Q_-$ ) will naturally transfer much of this dynamical energy to the disk gas. Thus, disk luminosity must increase and the disk itself will puff up. The scale height increase will be a function of the density of NCOs in the nucleus. Therefore, one prediction of our model is that among nuclei with similar supermassive black hole masses, those with denser stellar cusps, should generate more luminous AGN disks (see Paper II).

So far we have discussed low mass stars. However, we are interested in higher mass IMBH seeds. For simplicity let us assume a steep NCO mass function ( $dN/dM \propto M^{-3}$ ) with two mass bins. The low mass population NCOs are  $0.6 M_\odot$  stars ( $N_l$  in number); thus the high mass NCO population is  $10^{-3} N_l \times 10 M_\odot$  stellar mass black holes. For a total initial disk NCO mass of  $10^3 (10^4) M_\odot$ , the distribution is  $1.65 \times 10^3 (10^4)$  low mass stars and  $1 (10)$  stellar mass black holes. Alexander, Begelman & Armitage (2007) show that a low mass population of stars will diffuse out of the disk more than the high mass population of stars and in fact damp the orbits of the high mass stars, as expected from equipartition. Thus, for small initial values of  $< e^2 >^{1/2}$  among disk NCOs, we expect potential IMBH seeds in AGN disks to evolve to even smaller eccentricities than the equilibrium value of  $e \sim 0.01$  calculated above. Recall that small eccentricities imply large  $\sigma_{\text{coll}}$ , allowing IMBH seedlings to grow rapidly via collisions.

### 3.3 Binary mergers in the disk

Depending on the recent star formation history of a given galactic nucleus, massive binaries are likely to be rare in AGN disks. However, if there is even *one* in the initial AGN disk, it will have the largest collisional cross-section of any disk NCO and should undergo the largest number of interactions (e.g. Portegies-Zwart et al. 1999; Fregeau et al. 2004). A massive binary, if present, is therefore the most likely IMBH seed and should arise frequently enough to be of astrophysical interest (for similar ideas concerning planetesimal growth through binary-single interactions in a protoplanetary disk see e.g. Perets (2011)). In this section, we briefly consider some of the issues involved in binary mergers in an AGN disk and we contrast the merger efficiency with that found in star clusters. For ease of comparison we consider the  $50 M_\odot + 10 M_\odot$  binary from (Miller & Hamilton 2002).

An unequal mass binary ( $M > m$ , separation  $a_{\text{bin}}$ ) in the disk is considered hard if its binding energy ( $GMm/a_{\text{bin}}$ ) is greater than the kinetic energy ( $m\sigma^2 = m(\bar{e}^2 + \bar{i}^2)v_k^2$ ) of a typical interacting NCO. The collisional cross-section of such a binary is given by (Perets 2011)

$$\sigma_{\text{coll}} \approx \pi a_{\text{bin}}^2 \left( \frac{v_c}{v_\infty} \right)^2 \left( \frac{r_N / 10^6 \text{cm}}{a_{\text{bin}} / 2.14 \times 10^8 \text{cm}} \right) \quad (18)$$

where  $v_c = (G/\mu(Mm/a_{\text{bin}}))^{1/2}$  is the critical velocity separating hard and soft binaries, with  $\mu = (M_{\text{bin}} \times M_N)/(M_{\text{bin}} + M_N)$  is the reduced total mass of the binary ( $M_{\text{bin}}$ ) and interacting NCO ( $M_N$ ). The time to merge for a binary of reduced mass  $\mu = mM/(M+m)$  where  $M > m$ , semi-major axis  $a_{\text{bin}}$  and eccentricity  $e_{\text{bin}}$  is

$$\tau_{\text{merge}} \approx 3 \times 10^8 M_\odot^3 (\mu M^2)^{-1} (a_{\text{bin}}/R_\odot)^4 (1 - e_{\text{bin}}^2)^{7/2} \text{yr} \quad (19)$$

and the typical semi-major axis separation for a merger time of  $\tau_6 \text{Myr}$  (assuming  $e_{\text{bin}} \sim 0$ ) is

$$a_{\text{bin}} \approx 3 \times 10^{11} \tau_6^{1/4} M_{50}^{1/2} m_{10}^{1/4} \text{cm} \quad (20)$$

where  $M_{50}, m_{10}$  are the masses in units of  $50M_\odot$  and  $10M_\odot$  respectively (Miller & Hamilton 2002). However, the above discussion neglects the gas disk.

Baruteau et al. (2011) carried out hydrodynamic simulations of a binary in an AGN disk and found that binaries harden rapidly due to interaction with their own migratory spiral wakes. Baruteau et al. (2011) also found that  $e_{\text{bin}}$  is damped rapidly with inward migration. The rate of binary hardening ( $\dot{a}_{\text{bin}}$ ) scales with the disk surface density such that  $a_{\text{bin}}/\dot{a}_{\text{bin}} \ll \tau_{\text{bin}}$ , the binary migration timescale. Massive binaries with initial separation  $a_{\text{bin}} \sim 0.3R_H$ , end up at half this separation within 10 orbits of the supermassive black hole, where  $R_H$  is the Hill radius  $= (q/3)^{1/3}a$ , with  $a$  the semi-major axis of the binary center of mass and  $q$  the mass ratio of the reduced mass binary to the supermassive black hole. For a constant rate of hardening ( $\dot{a}_{\text{bin}}$ ), a  $M_{50}, m_{10}$  migrating binary separated by  $\sim 3 \times 10^{11} (10^{13}) \text{cm}$  (or  $3R_\odot (2 \text{ AU})$ ) at  $10^5 r_g$  will merge in  $< 0.1(2) \text{Myrs}$ , which is a very small fraction of the AGN disk lifetime. So, because of the presence of a gas disk, it is easier to harden binaries in AGN disks than in star clusters.

Binaries will also encounter field NCOs in the disk as they migrate. This can either result in hardening to merger or disruption. The probability that a binary is disrupted per unit time is  $1/t_{\text{dis}}$  where

$$t_{\text{dis}} = \frac{9|E|^2}{16\sqrt{\pi}\nu G^2 m^4 \sigma} \left( 1 + \frac{4m\sigma^2}{15|E|} \right) \left[ 1 + \exp\left( \frac{3|E|}{4m\sigma^2} \right) \right] \quad (21)$$

where the field NCO has mean mass  $m$  and velocity dispersion  $\sigma$ ,  $\nu$  is the number density of field NCOs,  $E = -GMm/a_{\text{bin}}$  is the binding energy of the binary, and the average energy change per interaction is  $\sim -0.2m\sigma^2$  (Binney & Tremaine 1987). In stellar clusters  $\sim 10^2$  field interactions are required to harden massive binaries to merger (Miller & Hamilton 2002), where the relative velocities at close encounters are approximately the velocity dispersion ( $\sim 50 \text{kms}^{-1}$ ) in star clusters. In AGN disks, the number of interactions required to harden a binary to merger depends on the mean eccentricity ( $\bar{e}$ ) of the field NCO orbits. The probability of binary disruption ( $1/t_{\text{dis}}$ ) increases as  $\bar{e}$  increases. However, for already hard binaries (large  $|E|$ ), fewer interactions (of energy  $-0.2m\sigma^2$ ) are required for merger in gas disks. Thus, hard binaries will continue to harden due to inward Type I migration (Baruteau et al. 2011) and due to interactions with field NCOs in the disk.

For a  $60M_\odot$  unequal mass binary ( $M_{50}, m_{10}$ ) at  $10^5 r_g$  interacting with a population of NCOs having  $\bar{e} = 0.01$ , the

energy per binary interaction ( $-0.2m\sigma^2$ ) is  $\sim 4\%$  the typical interaction energy in stellar clusters (where  $\sigma \sim 50 \text{km s}^{-1}$ ). So for highly damped NCO orbits, binary interactions are likely to be 'soft'. The number of interactions depends on the disk NCO surface density. If this binary is already hard, it could merge within a few orbits, i.e. orders of magnitude faster than binary merger timescales in stellar clusters.

## 4 NCO MIGRATION & COLLISIONS

The gas in the AGN disk exerts a net torque on NCOs. This means that individual NCOs (and IMBH) will migrate within the gas disk, enhancing the probability of collision and merger. Migration is mostly inward in disks, although sometimes outward. This means that IMBH can migrate into new regions of the disk, in search of new NCO 'victims'. The IMBH feeding zone (approximately  $a \pm 4R_H$ ) moves with the migrating IMBH. This is analogous to a giant planet core continuing to collide with planetesimals as it migrates through a protoplanetary disk Alibert et al. (2004). Migration can stall in disks, leading to a pile-up (overdensity of disk NCOs). In this case rapid merger leading to rapid IMBH growth may occur, by analogy with pile-up in protoplanetary disks (Horn et al. 2012). A detailed calculation of this scenario will be carried out in future work. NCOs and IMBH may also migrate onto the central supermassive black hole, just as protoplanets may migrate onto a central star. So the issue of IMBH survival in AGN disks parallels the survival of giant planets in protoplanetary disks.

### 4.1 Type I IMBH migration

NCOs with mass ratios  $q \leq 10^{-4}$  of the mass of the central supermassive black hole will undergo migration in the disk analogous to Type I protoplanetary migration, on a timescale of (Paardekooper et al. 2010)

$$\tau_1 = \frac{1}{N} \frac{M}{q\Sigma r^2} \left( \frac{H}{r} \right)^2 \frac{1}{\omega} \quad (22)$$

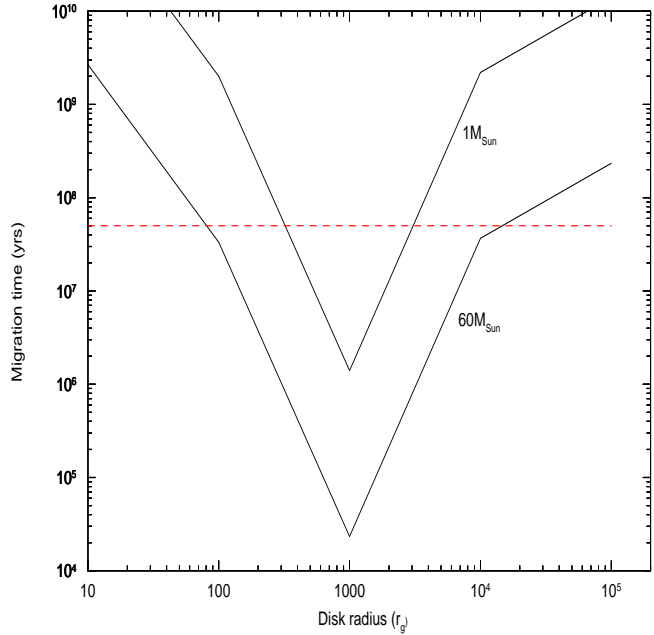
where  $M$  is the central mass,  $q$  is the ratio of the satellite (NCO) mass to the central (supermassive black hole) mass,  $\Sigma$  is the disk surface density,  $H/r$  is the disk aspect ratio and  $\omega$  is the satellite angular frequency. The numerical factor  $N$  depends on the ratio of radiative to dynamical timescales and is a function of the power-law indices of  $\Sigma, T$  and entropy (Lyra et al. 2010; Paardekooper et al. 2010). Note that the Type I migration timescale decreases with increasing migrator mass at a given radius so more massive NCOs will migrate more quickly at a given disk radius. Binary NCOs face exactly the same torques and will migrate on the same timescale, but  $q$  and  $r$  in eqn. 22 are replaced with, the ratio of the reduced mass of the binary to the supermassive black hole and  $a$ , the location of the binary center of mass in the AGN disk respectively.

Sirko & Goodman (2003) model an AGN disk including all the parameters we require for calculation of migrator timescales as a function of radius around a  $10^8 M_\odot$  supermassive black hole. Although in principle Sirko & Goodman (2003) model a disk out to  $10^7 r_g$ , they regard their disk as effectively truncating at  $\sim 10^5 r_g$ . This disk also requires a

constant mass accretion rate ( $\dot{M}$ ) and a constant disk viscosity ( $\alpha$ ) at all radii over the disk lifetime, which are obvious simplifications. Nevertheless using the simple AGN disk of Sirko & Goodman (2003) as our disk model, we can estimate migration timescales semi-analytically. Figure 3 shows the Type I migration timescales of a fiducial  $1M_{\odot}$  NCO (upper curve) and a  $60M_{\odot}$  IMBH seed (lower curve) as a function of disk location. The curves in Fig. 3 are generated by choosing  $N \sim 3$  in eqn.22 and assuming that  $\Sigma$  and  $H/r$  have the form of the curves in Fig. 2 of Sirko & Goodman (2003). Also marked in Fig. 3 is an approximate AGN lifetime of 50Myrs (red dashed line). Evidently, substantial changes of NCO orbital radius can occur even for low mass NCOs in the inner disk ( $< 10^4 r_g$ ) over the AGN lifetime (few  $\times 10$ Myr). Larger mass migrators (stellar mass black holes, binaries, large mass stars or seed IMBH) are likely to have migration timescales roughly comparable with the AGN disk lifetime. Therefore, as IMBH seedlings grow in mass they should migrate in the disk and encounter low mass NCOs at low relative velocities. If migration stalls for a given NCO, either inwards at small disk radii, or outwards at large disk radii, interactions are possible at incredibly low relative velocities ( $v_{\infty}$ ). Note that spiral density waves from migrating NCOs should not strongly perturb the NCO migrations or their orbits (Horn et al. 2012). Here we assume that seed IMBH migrate independently. Of course, resonant capture can occur, both between IMBHs and NCOs and between multiple IMBHs, analogous to resonances between Jupiter and its moons, or between Neptune and Pluto. Given the low mass ratios and high migration speeds, an assumption of independent migration seems reasonable, but future simulations involving multiple migrators are required to test this assumption.

Because we expect NCOs in the AGN disk to migrate differentially, we expect the migrators to encounter each other. As large mass NCOs migrate inwards across the orbits of less-massive NCOs, if the gas has damped  $\bar{\epsilon}$  (see §3.1 above) the relative velocities ( $v_{\infty}$ ) will be low and the collision cross-section (with gravitational focussing) will be large. However, different large mass NCOs will have different outcomes from multiple interactions. Large stars could have a shorter life expectancy if there are many NCO interactions increasing the odds of merger and supernova. Stellar mass black holes and IMBH seedlings will tend to shred low mass main sequence or giant stellar NCOs as they migrate inwards, although they can swallow compact NCOs whole as tidal forces shred compact objects only after the compact object crosses the innermost stable circular orbit (ISCO). Inward migrating binaries will tend to harden and scatter lower-mass NCOs until merger (see below).

Although a majority of Type I migration is directed inwards, Type I migration can also occur outwards in protoplanetary disks. For eccentricities  $e < 0.02$ , migration may be outwards rather than inwards (Bitsch & Kley 2010). Migration can also stall on both inward and outward migrations, depending on the temperature and density of the adiabatic disk (e.g. Veras & Armitage 2004; Lyra et al. 2010). So, as we consider a population of NCOs migrating in an AGN disk and interacting with each other, we do so with the caveat that a fraction of the NCOs may be migrating outwards, or stalled. From equation 22, the migration timescale gets longer at small disk radii ( $r$ ), where  $\Sigma$  de-



**Figure 3.** Estimates of Type I migration timescales for  $1M_{\odot}$  NCOs (upper curve) and  $60M_{\odot}$  IMBH seedlings (lower curve) as a function of radius in the AGN disk modelled by Sirko & Goodman (2003). Also shown (red dashed horizontal line) is a fiducial AGN lifetime of 50Myrs. Most disk NCOs should live in the outer disk ( $> 10^4 r_g$ ) for an NCO population that grows as  $r^2$ . Note that migration is fastest around  $10^3 r_g$  in the disk, where  $\Sigma$  is largest and the disk is thinnest ( $H/r$  is smallest). There is a possibility of NCO pile-up due to migration at smaller disk radii, as the co-rotating mass of gas drops dramatically in the disk interior.

creases and ( $H/r$ ) increases due to disk heating. Migration may even stall or cease at small disk radii, particularly for large migrator masses, when the co-rotating disk mass becomes less than the migrator mass (e.g. Syer & Clarke 1995; Armitage 2007). However, conditions in the hot inner disk may be dramatically different to the outer disk. If there is an abrupt transition to an optically thin accretion region, or a disk truncation or cavity, migration will stall. In this case, NCO pile-up can occur, potentially leading to mergers and ejections. Recent N-body simulations of protoplanetary disks suggest that migrator pile-up in regions of the disk where inward and outward torques balance results in very rapid merging (Horn et al. 2012). If migrator pile-up occurs in AGN disks, it could favour the rapid building of IMBH seedlings. A stalled IMBH seed can merge with and scatter piled-up NCO migrators. Using a simple equipartition of energy, an IMBH seedling of mass  $M$  stalled at  $10^2 r_g$  in an AGN disk with  $\bar{\epsilon}, \bar{i} \sim 0.01$  could scatter in-migrating NCOs ( $m$ ) to  $\sigma \sim (M/m)400 \text{ km s}^{-1}$ . At such hypervelocities, small mass NCOs can be ejected into a galactic halo (see Paper II).

If disk NCOs migrate inwards, their rate of migration decreases in the inner disk ( $< 10^3 r_g$ ) as the disk surface density and co-rotating disk mass drops (see also Fig. 3 above). Although conditions in the innermost AGN disk are dramatically different from those in the outskirts, it is useful to consider the possibility of NCO pile-ups. Evidently, if the AGN inner disk truncates at some radius and then becomes a geometrically thick, optically thin advection flow, NCO



migrators will stall near the disk truncation radius. As NCOs build up over time, the chances of interaction increase and hypervelocity scatterings become likely. The conditions may be similar to the migration trap for N protoplanets in protoplanetary disks, where rapid merger is possible (Horn et al. 2012). In this case, IMBH could form in the inner disk with distinctive observational signatures (see Paper II for details). Of course, migration traps can also occur as a result of out-migration.

A majority of NCOs in galactic nuclei will have orbits that do not coincide with the plane of the AGN disk ( $i \geq 0.05$ ) and will instead punch through the disk periodically. These NCOs can interact with each other, the disk and the migrating NCOs in the disk. NCOs on orbits with small radii will eventually decay into the plane of the disk over the AGN disk lifetime (Artymowicz et al. 1993a), growing the NCO disk population ( $\dot{N}_+$ ), particularly at small disk radii. Resonant relaxation and the Kozai mechanism will also allow non-disk NCOs to trade eccentricity with inclination and migrate into the disk over time (e.g. Rauch & Tremaine 1996; Subr & Karas 2005; Chang 2009). On the other hand, NCO interactions within the disk that lead to ejection should keep ejected NCOs at relatively low inclinations, thereby increasing the probability of re-capture by the disk (growing  $\dot{N}_+$ ). A disk capture rate of  $\sim 10^{-4}$  of the non-disk NCO population over the lifetime of the AGN disk, corresponds to the capture of  $\sim 10^3 M_\odot$  of non-disk NCOs mostly in the inner AGN disk, over the  $\sim 50$  Myr disk lifetime around a  $10^8 M_\odot$  supermassive black hole.

Unlike protoplanets in disks around stars, a large number of NCOs in AGN disks should have retrograde orbits. The behaviour of retrograde NCOs will depend on the efficiency of angular momentum transfer between the NCO and the disk gas. On one hand, if the coupling between NCO and gas is strong, the retrograde NCO rapidly loses angular momentum and falls into the central supermassive black hole very quickly. On the other hand, if the coupling is very weak, the disk gas can move fast enough past the NCO that the NCO can persist in the disk for a long time without migrating. In this case, prograde NCOs will migrate and encounter retrograde NCOs.

## 4.2 Type II IMBH migration

An IMBH that grows large enough by accreting gas can open a gap in the AGN disk (Syer & Clarke 1995). This phenomenon is analogous to gap opening by massive planets in protoplanetary disks (Armitage 2010). For typical disk parameters ( $H/r \sim 0.05$ ,  $\alpha = 0.01$ ), the mass ratio required to open a gap is  $q \sim 10^{-4}$ . However, there is a strong dependency on disk viscosity for both the profile and depth of the gap (Crida et al. 2006; Muto et al. 2010). To open a gap in the disk requires low disk viscosity (Crida et al. 2006)

$$\alpha < 0.09 q^2 \left( \frac{H}{r} \right)^{-5} \quad (23)$$

where  $\alpha$  is the disk viscosity parameter (Shakura & Sunyaev 1973). In the AGN disk modelled by Sirko & Goodman (2003),  $H/r \sim 0.05$  on average between  $10^2 - 10^5 r_g$  and  $\alpha = 0.01$  is fixed. An IMBH with  $q > 3 \times 10^{-4}$  ( $> 3 \times 10^4 M_\odot$ ) will clear a gap in this disk. The gap will close by pressure if  $(H/r) > (q/\alpha)^{1/2}$  and by accretion if  $(H/r) > (q^2/\alpha)^{1/5}$

(Syer & Clarke 1995). Thus, a gap opened by a  $3 \times 10^4 M_\odot$  IMBH in the AGN disk modelled by Sirko & Goodman (2003) will be closed by pressure and/or accretion in the outermost and innermost parts of the disk where  $(H/r)$  and  $\alpha$  are large. In a more viscous type of accretion flow ( $\alpha \geq 0.1$ , e.g. advection dominated), an IMBH might not open a gap in the disk. Whether an IMBH opens a gap will have major implications for observational signatures in AGN, but we defer that discussion to Paper II.

An IMBH that opens a gap will tend to migrate on the viscous disk timescale (Type II migration) given by

$$\tau_{II} = \frac{1}{\alpha} \left( \frac{h}{r} \right)^{-2} \frac{1}{\omega}. \quad (24)$$

From eqn. 24, for  $\alpha \sim 0.01$  and  $H/r \sim 0.05$  (approximately the conditions across the AGN disk modelled by Sirko & Goodman (2003)), the Type II migration timescale is  $\sim 10^4 \times$  the orbital timescale. So, at  $10^4 (10^5) r_g$ , the Type II migration timescale is  $\sim 1(30)$  Myrs. Evidently a gap-opening IMBH can migrate on timescales shorter than the AGN lifetime across the disk. We therefore have to ask whether any gap-opening IMBH will survive the AGN disk? This is analogous to a major problem encountered in protoplanetary disk theory. The migration of some gap-opening migrators must somehow stall before accretion onto the central mass, otherwise no Jupiter-mass planets would be observed. One solution to this problem is that Type II migration can stall once the co-rotating disk mass is less than the migrator mass. This condition could arise due to disk drainage onto the supermassive black hole, or a change in the surface density profile of the disk. In the Sirko & Goodman (2003) disk, this radius is  $\sim 10^4 r_g$  for a  $3 \times 10^4 M_\odot$  IMBH. Once Type II migration stalls, it can resume but at a much slower rate, once the migrators' angular momentum is exported to the local disk (e.g. Syer & Clarke 1995; Armitage 2010).

If we simply assume the IMBH can undergo Type II in-migration without stalling, the IMBH will collide with NCOs at radii interior to its starting position  $10^4 (10^5) r_g$  before accreting onto the supermassive black hole in  $1(30)$  Myrs. Ignoring gas accretion, the IMBH can swallow up to 5%(50%) of the uniformly distributed disk NCO population if it starts migrating at  $10^4 (10^5) r_g$ . So, IMBH growth via NCO merger can be as much as  $\sim 5000 M_\odot / 30$  Myr (starting at  $10^5 r_g$  and  $10^4$  NCOs in the disk). This growth rate is due to NCO mergers only and does not include growth due to gas accretion (see §5.2 below). If the IMBH stalls permanently at  $\sim 10^4 r_g$ , only a small number of remaining migrators ( $\sim 2\%$  of the remaining disk NCO population) migrate inwards to merge with the IMBH within 50 Myrs.

To sum up, for a powerlaw stellar mass function ( $\sim M^{-3}$ ), most NCOs should be low mass stars ( $< 1 M_\odot$ ) with a small fraction of compact NCOs (mostly white dwarfs, some neutron stars and stellar mass black holes). The largest mass NCOs (and seeds for IMBHs) are likely to be small in number, and consist of stellar mass black holes or massive binaries. IMBH seedlings will undergo Type I migration in the disk. IMBH migration means that they can maintain a feeding zone of disk NCOs as they 'catch up' with the much more slowly migrating low mass disk NCOs. This migration of the feeding zone is precisely analogous to the situation expected for migrating giant planets (e.g. Alibert et al. 2004).



Some of the classic problems of protoplanetary migration (e.g. how to stop migrators from accreting onto the central object, or how to get migrators moving once stalled) will also apply to IMBH seeds in AGN disks. Nevertheless, low relative velocity encounters due to migration will result in large NCO collision cross-sections and will help grow IMBH seeds via core accretion.

## 5 A MODEL OF IMBH GROWTH IN AGN DISKS

In this section, we shall draw together much of the above discussion and construct a simple model of IMBH growth in AGN disks. The starting point for our model is a stellar mass black hole. This  $10M_\odot$  black hole can accrete gas from the AGN disk, migrate within the disk and collide with disk NCOs. We follow the approach of models of giant planet growth in protoplanetary disks (Pollack et al. 1996; Alibert et al. 2004). In section 5.1 we discuss the growth of the IMBH seedling as a result of collision with disk NCOs within the IMBH feeding zone. In section 5.2 we discuss the growth of the IMBH seedling as the result of gas accretion.

### 5.1 The parallel with 'core accretion'

We considered a simple model of the growth of a  $10M_\odot$  IMBH seed embedded in an AGN disk around a  $10^8M_\odot$  black hole. The disk NCO initial population is  $(10^3)10^4M_\odot$ , with a mass function of  $dN/dM \propto M^{-3}$  (as discussed above mostly  $0.6M_\odot$  stars). We assume that the IMBH seed 'feeds' on NCOs within its accretion zone, given by  $a \pm \delta a$  where  $\delta a \sim 4R_H$ , analogous to the feeding zone of giant planet cores (Pollack et al. 1996). The maximum (isolation) mass that the IMBH seed can attain by feeding on all the NCOs within its accretion zone is

$$M_{\text{ISO}} = M_I + 16\pi a^2 \Sigma_0 \left(\frac{q}{3}\right)^{1/3} \quad (25)$$

where  $\Sigma_0$  is the mean initial NCO surface density,  $q$  is the mass ratio of the IMBH seed to the supermassive black hole. If there are  $10^4M_\odot$  NCOs initially in the disk around a  $10^8M_\odot$  black hole, the mean initial NCO surface density is  $\Sigma_0 \sim 3.5\text{g/cm}^2$ . For an IMBH seed of  $M_I = 10M_\odot$ , eqn. 25 implies  $M_{\text{ISO}} \sim 10 + 9(900)M_\odot$  at  $10^4(10^5)r_g$ . So, in principle, a stellar mass black hole in the outer disk could grow to many times its original mass just by accreting low mass disk NCOs. This process is analogous to the growth of giant planet cores by planetesimal accretion (Pollack et al. 1996; Armitage 2010).

Assuming small eccentricities ( $\langle e^2 \rangle^{1/2} = \Delta a/a$ ) for disk NCOs, we are in a shear-dominated regime and the rate of mass growth of the IMBH seed may be approximated by the form of giant planet core growth as (Armitage 2010)

$$\frac{dM}{dt} = \frac{9}{32} \frac{(\Delta a)^2}{\langle i^2 \rangle^{1/2} a R_H} \nu \Sigma_0 \Omega \sigma_{\text{coll}} \quad (26)$$

where  $\langle i^2 \rangle^{1/2}$  is the rms inclination for the NCO distribution,  $\nu$  is the relative local overdensity of disk NCOs and  $\sigma_{\text{coll}}$  is the collision cross-section as given by eqn. 1. Thus, if  $\Delta a = \langle e^2 \rangle^{1/2} a$  and if we assume  $\langle e^2 \rangle^{1/2} \sim 2 < i^2 \rangle^{1/2}$ ,

with  $v_\infty \sim \sigma \approx \langle e^2 \rangle^{1/2} v_k$ , the rate of IMBH seed growth via core accretion within its feeding zone is

$$\frac{dM}{dt} = \frac{9}{8} \frac{\nu \Sigma_0 \Omega \pi r_p}{e(q/3)^{1/3}} \frac{2GM}{v_k^2} \quad (27)$$

where  $r_p$  is the periastron. The periastron for compact object collisions with an IMBH seed is  $r_p \sim R_\odot$  (Miller & Hamilton 2002). Substituting into eqn. 27, where we assume  $\langle e^2 \rangle^{1/2} \sim 0.01$  is the equilibrium eccentricity, we find that for a  $10M_\odot$  IMBH seed at  $\sim 2 \times 10^4 r_g$  with  $\Sigma_0 = 3.5\text{gcm}^{-2}$  and  $\nu = 1$ , then  $dM/dt \sim 10^{-7}M_\odot/\text{yr}$ , which is approximately half the Eddington rate of growth. This is a very high accretion rate for a black hole, exceeding inferred accretion rates from gas disks in most Seyfert AGN (McKernan et al. 2007). Of course  $\Sigma_0$  is the average surface density assuming a uniform distribution of disk NCOs and that the mass ratio of disk NCOs to gas in the disk is  $\sim 1\%$ . If, instead the mass ratio is a factor of a few larger, or the surface density distribution of NCOs is non-uniform, the accretion rate of disk NCOs by IMBH can be substantially *super-Eddington*. Equally, if gas damping is more efficient than outlined above so that equilibrium is reached at  $\bar{e} < 0.01$  (e.g. due to a lower ratio of cusp population to disk NCOs), we could also reach super-Eddington rates of IMBH growth via mergers. From our earlier discussion, it is easy to envisage regions of the disk where NCOs tend to pile-up, leading to non-uniform distributions of disk NCOs. For example, if there is a region of the disk where inward and outward torques balance (as in the scenario outlined by Horn et al. (2012)), or migration stalls due to a change in the aspect ratio, or the mass of co-rotating gas drops. In these cases, we could write  $\Sigma_0 \sim \nu 3.5\text{gcm}^2$ , where  $\nu$  is an overdensity factor (which could locally be  $> 100$  in a pile-up scenario such as in Horn et al. (2012) and lead to highly super-Eddington growth). Note that a growing IMBH seedling avoids problems in protoplanetary coagulation theory, such as fracturing and sticking efficiency. For IMBH seedlings, nearby objects will either be captured (at high efficiency) or they will escape.

Figure 4 shows the analytic growth of a  $10M_\odot$  stellar mass black hole (IMBH seed) and a  $100M_\odot$  IMBH in an AGN disk around a  $10^8M_\odot$  supermassive black hole. The lower curve in each case corresponds to  $\nu = 1$  (no overdensity, fiducial numbers) and the upper curves corresponds to a moderate over-density  $\nu \sim 5$  of disk NCOs (or equivalently, a slight overdensity,  $\nu = 2$ , and a moderately lower mean eccentricity  $\bar{e} = 0.004$ ). The fiducial mass doubling time for a black hole accreting gas at the Eddington rate (assuming 10% efficiency) is  $4 \times 10^7$  yrs. In the case of IMBH seeds growing via collisions in AGN disks, and assuming a gas accretion rate of  $\sim$  Eddington, the total growth rates are  $\times 1.5(3.5)$  Eddington for  $\nu = 1(5)$ . At  $3.5 \times$  Eddington growth rates, the mass doubling time could be as little as  $\sim 11\text{Myrs}$ . Once the IMBH reaches its isolation mass, it will then grow via accretion from the gas disk, at a much slower rate (Eddington or a fraction thereof). However, with an increased mass, the IMBH will have a shorter Type I migration timescale (see §4 above). Thus, in  $\sim 11\text{Myrs}$ , the IMBH will have migrated inwards or outwards in the disk and the size of the feeding zone will have grown. So, a super-Eddington mode of accretion via collisions could continue (dashed curve in Fig. 4). This process is analogous to the migration of giant

planet cores and their feeding zones in protoplanetary disks (Alibert et al. 2004).

## 5.2 IMBH and gas accretion: runaway growth

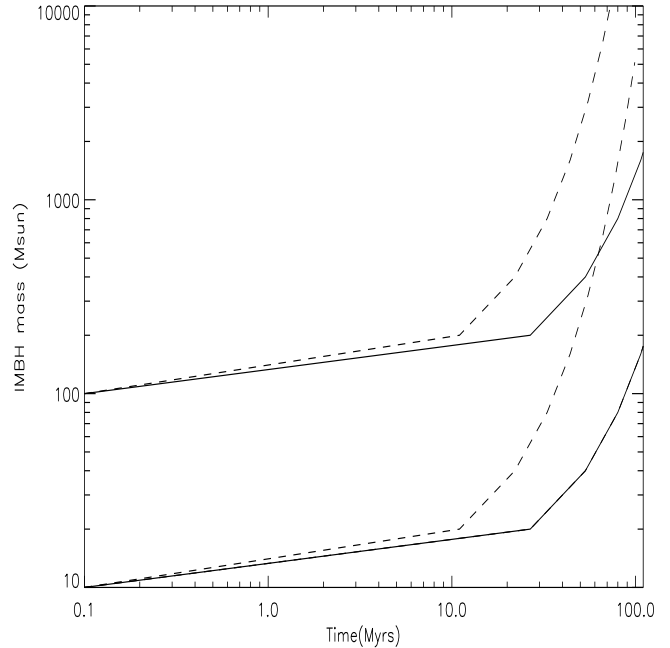
A big difference between IMBH growth in stellar clusters and in AGN disks is that in the latter, gas can damp orbits quite effectively. So mergers tend to be more frequent, and the IMBH seeds can continuously accrete dense gas. Thus, we expect IMBH growth in AGN disks to involve growth by merger (as in stellar clusters) but we also expect growth by gas accretion. Torques from the gas will cause the IMBH to migrate and enter new feeding zones, analogous to the situation in protoplanetary disks (Alibert et al. 2004). If the IMBH grows large enough ( $q = 10^{-4}$  or  $10^4 M_\odot$  around a  $10^8 M_\odot$  supermassive black hole), the IMBH can open a gap in the gas disk and the rate of gas accretion will drop. Note that although we concentrate on building a single IMBH, multiple IMBH seedlings ( $10 - 100 M_\odot$ ) are likely to appear in the disk (assuming  $dM/dt \propto M^{-3}$ , see discussion above).

One problem will be in preventing an IMBH from migrating onto the supermassive black hole. Outward migration and the stalling of migration due to a drop in disk surface density or a change in the disk aspect ratio are possible solutions, but as with protoplanetary disk theory, this theoretical problem is complicated and remains unsolved for now. A sufficiently massive gap-opening IMBH ( $\geq 10^4 M_\odot$  around a  $10^8 M_\odot$  supermassive black hole) will grow if the AGN disk is particularly long-lived ( $> 50$  Myrs), or if there is a large local disk NCO overdensity ( $\nu$ ), or if gas damping is particularly efficient so that equilibrium eccentricity is  $\bar{e} < 0.01$ . Alternatively, an IMBH which survives a period of AGN activity could grow to gap-opening size via the mechanisms outlined here, during a later, independent AGN phase. Earlier in the history of the Universe (at  $z \sim 2$ ), the time between individual AGN phases should be much smaller (e.g. Vittorini et al. 2005; Doherty et al. 2006; Shankar et al. 2009; McKernan et al. 2010b). So, if they survive, large mass (gap-opening) IMBH could grow rapidly in galactic nuclei over a few 100 Myrs and observational signatures of IMBH in galactic nuclei may be common at higher redshift (see Paper II).

Of course, as the gas disk is consumed or blown away, other mechanisms will come into play. For planetesimals in a late-stage protoplanetary disk, planet-planet interactions, the Kozai mechanism or resonant relaxation can increase  $\bar{e}$ ,  $\bar{i}$  (e.g. Rauch & Tremaine 1996; Subr & Karas 2005; Chang 2009). By analogy with protoplanetary disks, such mechanisms will certainly apply in the late stages of an AGN disk when most gas has been drained. However, we do not consider these mechanisms in more detail here since damping due to dense gas disk should dominate such effects (see Paper III for further discussion).

## 6 CONCLUSIONS

We show that it is possible to efficiently grow intermediate mass black holes (IMBH) from stars and compact objects within an AGN disk. Nuclear cluster objects (NCOs) in the AGN disk are subject to two competing effects: orbital excitation due to cusp dynamical heating and orbital damping



**Figure 4.** The growth over time of a  $10 M_\odot$  IMBH seed and a  $100 M_\odot$  IMBH located  $2 \times 10^4 r_g$  from a  $10^8 M_\odot$  supermassive black hole in an AGN disk. The solid (dashed) curves show IMBH growth as it accretes disk NCOs within  $\leq 4 r_H$  at a rate  $\times 0.5(2.5)$  Eddington, assuming  $\bar{e} = 0.01$  and an initial NCO surface density  $\Sigma_0 \sim 3.5(18) \text{ g cm}^{-2}$ . The dashed curves could equivalently be generated with  $\Sigma_0 \sim 7 \text{ g cm}^{-2}$  and  $\bar{e} = 0.004$ . The gas accretion rate is assumed to be at the Eddington rate, for a total accretion rate of  $\times 1.5(3.5)$  Eddington and a mass doubling time of  $\sim 27(11)$  Myr respectively. We assume the IMBH migrates in the disk and moves its feeding zone, so that it may continue to undergo collisions with NCOs at up to super-Eddington rates (analogous to continued collision and migration in protoplanetary disks Alibert et al. (2004)). In order to clear out  $> 10^3 M_\odot$  of disk NCOs from this choice of initial disk location, *outward* migration is required. Once the IMBH reaches  $10^4 M_\odot$  we assume the disk NCO population has been cleared out and the IMBH opens a gap in the disk, accreting gas at an Eddington rate thereafter. In this picture, between 1/3 and 2/3 of the IMBH mass is actually due to gas accretion.

due to gas in the disk. For a simple, semi-analytic model we show that gas damping dominates such that equilibrium eccentricities of disk NCOs are  $e \sim 0.01$ . In this case IMBH seedling formation via NCO collision is more efficient in the AGN disk than in stellar clusters (the standard model for IMBH formation). If, as we expect, gas damping dominates then the dynamical heating of disk NCOs by cusp stars is transmitted to the gas disk. This is a new, additional source of heating of the outer disk that can help counter the well-known gravitational instability ( $Q \leq 1$ ) of the outer disk.

Stellar mass black holes and hard massive binaries are likely IMBH seeds. IMBH seedlings grow by collisions with disk NCOs within their feeding zone ( $a \pm 4 r_H$ ) at near Eddington rates, as well as via gas accretion. IMBH seedlings will migrate within the AGN disk and so continue to feed on disk NCOs as they accrete gas. If there are regions of modest over-density of NCOs in the disk, IMBH seedling growth via collisions can be super-Eddington and a  $10 M_\odot$  IMBH seed orbiting a  $10^8 M_\odot$  supermassive black hole can grow to

$\sim 300M_{\odot}$  in less than the fiducial AGN disk lifetime. An over-density of disk NCOs can occur in regions of the disk where e.g. outward torques and inward torques balance, or where the aspect ratio changes, or where IMBH migration stalls.

The largest IMBH will open gaps in AGN disks, analogous to giant planets in protoplanetary disks. Gap-opening IMBH are more likely to arise if: gas damping is very efficient (equilibrium disk NCO eccentricity is  $\bar{e} < 0.01$ ), or if the disk is long-lived ( $> 50\text{Myrs}$ ), or disk NCO surface density is moderately high ( $> 15\text{gcm}^{-2}$ ), or if there is an IMBH seedling which survived a previous AGN phase (analogous to the survival of planets in protoplanetary disks). Our model of IMBH growth in AGN disks strongly parallels the growth of giant planets in protoplanetary disks. We leave a discussion of model predictions, observational constraints and implications of efficient IMBH growth in AGN disks to Paper II.

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## REFERENCES

- Alexander R.D., Begelman M.C. & Armitage P.J. 2007, *ApJ*, 654, 907
- Alibert Y., Mordasini C. & Benz W., 2004, *A&A*, 417, L25
- Armitage P.J. 2007, *ApJ*, 665, 1381
- Armitage P.J. 2010, *Astrophysics of Planet Formation*, Cambridge University Press
- Artymowicz P., Lin D.N.C. & Wampler E.J., 1993a, *ApJ*, 409, 592
- Artymowicz P., 1993b, *ApJ*, 419, 166
- Bartko H. et al., 2009, *ApJ*, 697, 1741
- Begelman M.C. & Rees M.J., 1978, *MNRAS*, 185, 847
- Baruteau C., Cuadra J. & Lin D.N.C., 2011, *ApJ*, 726, 28
- Binney J. & Tremaine S., 1987, *Galactic Dynamics*, Princeton University Press.
- Bitsch B. & Kley W., 2010, *A&A*, 523, 30
- Chang P., 2009, *MNRAS*, 393, 224
- Cresswell P., Dirksen G., Kley W. & Nelson R.P. 2007, *A&A*, 473, 329
- Crida A., Morbidelli A. & Masset F. 2006, *Icarus*, 181, 587
- Doherty M., Bunker A., Sharp R., Dalton G., Parry I. & Lewis I. 2006, *MNRAS*, 370, 331
- Fregeau J.M., Cheung P., Portegies-Zwart S.F. & Rasio F.A. 2004, *MNRAS*, 352, 1
- Goodman J. & Tan J.C., 2004, *ApJ*, 608, 108
- Hopkins P.F. & Hernquist L., 2006, *ApJS*, 166, 1
- Horn B., Lyra W., Mac Low M.-M. & Sándor Z., 2012, *ApJ*, 750, 34
- Islam R.R., Taylor R.R. & Silk, 2004, *MNRAS*, 354, 427
- Jiang Y.-F., Greene J.E., Ho L.C., Xiao T. & Barth A.J., 2011, *ApJ*, 742, 68
- Kawawatu N., Umemura M. & Mori M., 2003, *ApJ*, 583, 85
- King A.R., 2009, *MNRAS*, 393, L41
- Kormendy J. & Richstone D., 1995, *ARA&A*, 33, 581
- Krolik J.H., 1999, *Active Galactic Nuclei*, Princeton University Press.
- Levin Y., 2007, *MNRAS*, 374, 515
- Lyra W., Paardekooper S.-J. & Mac Low M.-M., 2010, *ApJ*, 715, L68
- Madau P. & Rees M.J., 2001, *ApJ*, 551, L27
- Micic M., Holley-Bockelmann K. & Sigurdsson S., 2011, *MNRAS*, 414, 1127
- McKernan B., Yaqoob T. & Reynolds C.S., 2007, *MNRAS*, 379, 1359
- McKernan B., Ford, K.E.S. & Reynolds C.S., 2010b, *MNRAS*, 407, 2399
- McKernan B., Maller A. & Ford, K.E.S., 2010a, *ApJ*, 718, L83
- McKernan B., Ford, K.E.S., Lyra W., Perets H.B., Winter L.M. & Yaqoob T., 2011a, *MNRAS*, 417, L103
- McKernan B., Ford, K.E.S., Yaqoob T. & Winter L.M., 2011b, *MNRAS* 413, L24
- McKernan B., Ford, K.E.S., Lyra W., Perets H.B. & Winter L.M., 2012, *MNRAS*, in prep. (Paper II)
- Merritt D., 2010, *ApJ*, 718, 739
- Miller M.C. & Colbert E.J.M., 2004, *IJMPD*, 13, 1
- Miller M.C. & Hamilton D.P., 2002, *MNRAS*, 330, 232
- Miralda-Escudé, J. & Gould A., 2000, *ApJ*, 545, 847
- Morris, M. 1993, *ApJ*, 408, 496
- Muto T., Suzuki T.K. & Inutsuka S.-I., 2010, *ApJ*, 724, 448
- Nayakshin S. & Sunyaev R., 2007, *MNRAS*, 377, 1647
- Ostriker J.P., 1983, *ApJ*, 273, 99
- Paardekooper S.-J., Baruteau C., Crida A. & Kley W., 2010, *MNRAS*, 401, 1950
- Perets H.B., Gualandris A., Merritt D. & Alexander T., 2008, *MmSAI*, 79, 1100
- Perets H.B., 2011, *ApJ*, 727, L3
- Pollack J.B., Hubickyj O., Bodenheimer P., Lissauer J.J., Podolak M. & Greenzweig Y., 1996, *Icarus*, 124, 62
- Portegies-Zwart S.F., Makino J., McMillan S.L.W. & Hut P., 1999, *A&A*, 348, 117
- Portegies-Zwart S.F., Baumgardt H., Hut P., Makino J. & McMillan S.L.W. 2004, *Nature*, 428, 724
- Quinlan G.D. & Shapiro S.L., 1989, *ApJ*, 343, 725
- Rauch K.P. & Tremaine S., 1996, *NewA*, 1, 149
- Remillard R.A. & McClintock J.E., 2006, *ARA&A*, 44, 49
- Schödel R., Merritt D. & Eckart A. 2009, *Astron. Ap.*, 502, 91
- Sirko E. & Goodman J. 2003, *MNRAS*, 341, 501
- Shakura N.I. & Sunyaev R. 1973, *A&A*, 24, 337
- Shankar F., Weinberg D.H. & Miralda-Escude J. 2009, *ApJ*, 690, 20
- Subr L. & Karas V. 2005, *A&A*, 433, 405
- Syer D., Clarke C.J. & Rees M. 1991, *MNRAS*, 250, 505
- Syer D. & Clarke C.J. 1995, *MNRAS*, 277, 758
- Veras D. & Armitage P.J. 2004, *MNRAS*, 347, 613
- Vittorini V., Shankar F. & Cavaliere A. 2005, *MNRAS*, 363, 1376
- Ward W.R. & Hahn J.M. 1994, *Icarus*, 110, 95
- Winter L.M., Mushotzky R.F., Reynolds C.S. & Tueller J. 2009, *ApJ*, 690, 1322